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# Dynamic Modeling of PEM Fuel Cell Dedicated for transport Application

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*Abstract***— this paper propose a non-linear state-space dynamic model for planar PEM fuel cell dedicated for transport application. The model is based on both thermodynamic and electrical aspects. Then an analysis of this model by changing the parameters like hydrogen flow rate, pressure and membrane humidity. The model simulation results show the effect of this of different parameters.**

*Keywords*— *PEM fuel cell, hydrogen, hydrogen flow rate, membrane humidity, partial pressure*

## I. INTRODUCTION

Since the decrease of oil reserves, due to the massive exploitation of the fuel cell, the domain of energy becomes the focus of the public interest. In addition, the environmental pollution requires the development of new technologies of energy  $[1]$ .

In this paper, we study the proton exchange membrane fuel cell (PEMFC) which is one of new energy and its considered as the most appropriate type of fuel cell for replacing internal combustion engines. It uses a solid polymer membrane as electrolyte and usually works at average temperature (between 40 and 90°C). Compared to other types of fuel cells, PEMFC is more compact and **lightweight** because it generates a better volumic power and power-density. In addition, its operating temperature is less than  $100^{\circ}$ C, allowing rapid start-up. These facts and ability to rapidly change power output are some of characteristics that make PEMFC suitable for automotive power applications. Other advantages result from the solid nature of the electrolyte because with a solid electrolyte the sealing of anode and cathode gases is far easier, and therefore, less expensive to manufacture. It has also less problems with corrosion, which means a longer cell stack life [1].

There are several studies in the literature related to the PEMFC modeling. These studies aim at understanding the phenomena occurring in the fuel cell in order to make the appropriate control. A number of mathematical models have been developed for this purpose but they are mostly steadystate models which are typically used for component sizing [2-4], dynamic model [5,7] These models aim at understanding the phenomena occurring in the fuel cell in order to make the appropriate control.

We propose in this paper a non-linear state-space dynamic model for single cell of planar PEMFC structure dedicated for transport application that takes into account the influence of several parameters on the system behavior. Our model is based on both thermodynamic and electric aspects. On the one hand, the diffusion principle, which is drawn from [5], is used to describe pressures and flow rates evolutions. On the other hand, the electric principle is used to model the voltage and the current. The dynamic behaviors of voltage, current are stated with respect to gas flow rate in gas bulk, partial pressures and humidification rate.

The remainder of this paper is organized as follows: Section 2 explains the PEMFC principle and states some Known results and remarks, which are needed in the subsequent development. Section 3 presents a way to build the dynamic model of a single PEM cell. Simulation results and analysis are given in section 4, followed by conclusions in section 5.

## II. FUEL CELL PRINCIPLE

A fuel cell is an electrochemical device that converts the chemical of reactants (fuel and oxidant) directly into electricity energy. Water and heat are the only by-products if hydrogen is used as the fuel source. The principle of PEMFC is shown in Fig. 1[8].

At the anode, the hydrogen molecules is split into hydrogen ions (protons) and electrons. The hydrogen ions permeate across the membrane to the cathode while the electrons flow through an external circuit and produce electric power. Oxygen, usually in the form of air, is supplied to the cathode and combines with electrons and hydrogen protons to produce water. Electrons circulation between anode and cathode through an external circuit generates electricity. The reaction that takes place at the triple phase boundary zone (TPB).

A typical PEM fuel cell has the following reactions in [8]:

Anode: 
$$
H_2 \rightarrow 2H^+ + 2e^-
$$
  
\nCathode:  $\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O$   
\nOverall:  $H_2 + \frac{1}{2}O_2 \rightarrow H_2O + electricity + heat$ 



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Fig. 1. PEM fuel cell principle

## *A. Voltage output*

In the ideal situation (reversible), electrical work is equal to the Gibbs free e<mark>nergy release</mark>d  $\Delta g_{\overline{f}}$  [9]

$$
\Delta g_f = -2FE \tag{1}
$$

The electromotive force or reversible open circuit voltage E of the hydrogen fuel cell is given as

$$
E = E^0 + \frac{RT}{2F} \ln \left( \frac{P_{H_2} P_{O_2}^{1/2}}{P_{H_2O}} \right)
$$
 (2)

Where  $P_{H_2}, P_{O_2}$  and  $P_{H_2O}$  are reactant partial pressure, E is also called Nernst voltage.

Irreversibilities reduce the cells voltage. They are mainly activation loss, ohmic loss and concentration loss. The voltage is usually modeled in the steady state form [10]:

$$
V = E - iR_o - A \ln\left(\frac{i}{i_0}\right) - B \ln\left(1 - \frac{i}{i_l}\right) \tag{3}
$$

Where *i* is the current produced by the cell.  $R_o$  is the inherent resistance of the fuel cell.  $i_0$  is the exchange current, an important parameter of weighting the activity of catalyst reaction.  $i_l$  is the limiting current, at which the fuel is used up at rate equal to its maximum supply rate. *A* and *B* are coefficients. The second term in the equation represents the ohmic loss, the third term is activation loss and the fourth term represents the concentration loss.

## *B. Current output*

Neglecting the transit dynamics of the reactions, the relation between current and reactions can be expressed as [9]:

$$
i = 2FJ_{H_2}^b = 2FJ_{H_2O}^b = 4FJ_{O_2}^b
$$
 (4)

Where superscript *b* represents the gas flow into the outer surface of diffusion layer or water vapor production rate at cathode. The maximum current that a cell can output is limited by reactant supply rates, the area where reactions take place and ionic conductivity of electrolyte.

#### III. MODELING APPROACH

This paper deals with modeling the dynamic performances of a fuel cell focusing on both electrical and diffusion approaches. The dynamics of the PEMFC will be represented by non-linear differential equations transformed into an equivalent mathematical formulation: the state space representation. The method of state space domain exhibits great advantages especially in the study of control, parameter estimation and optimization. Besides, it presents a powerful tool for simulation.

As mentioned in section 1, we are interested on the planar fuel cell geometry. This structure is built by stacking together multiple single cell. Hence, the fuel cell stack is electrically a serial connection of these cells. It is basically this fact which motivates us to focus on a single cell modeling since in a serial connection the single cell voltage is added up  $[11]$ . In addition, we consider that the fuel cell lies in a mesoscopic scale.

For this reason, we may consider a single cell as a lumped volume of the stack. This enables us to assume an uniform variation of the temperature inside the single cell.

According to these hypothesis, we study a model based on diffusion and electrical principles. The PEMFC modeling process will be divided into three parts. In the first part, we establish relation between pressures and flow rates according to concentration and diffusion laws. Then, we set electrical relationships between voltage and current in the second part. The third part is devoted to the state-space formulation.

The first *input is the hydrogen* flow rate into the outer surface of diffusion layer influences the current output and so affects the reaction. Other inputs are partial pressures of reactants in gas bulks. They affect the diffusion processes as well as the Gibbs free energy and thus the voltage and current outputs. The fifth input is the humidification rate  $\lambda$  related to water management. Its role is to maintain the membrane humidity and to balance water usage/consumption in the system. Indeed, on one hand, if the humidification level of the membrane is too low, the proton transportation through the membrane will become difficult and this will lead to higher internal resistance. Another input that influences the humidification resistance is the temperature *T* which together



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with humidification rate influences the ohmic losses. This eventually affects voltage and current outputs. Finally, the output variables are voltage, current. we define in table I the input/output variables.





## *A. Diffusional approach*

In this part, we are going to use the diffusion principle in order to establish the expression of gas partial pressures  $P^{tpb}$  at TPB and gas consumption rate  $J^{tpb}$  at TPB with respect to both gas consumption rate  $J^b$  at gas bulk and  $\overline{P}^b$ . Indeed, in our planar geometrical model, we determine the diffusion flux due to the concentration difference between gas bulk and TPB zones according to Fick's first law:  $J = -D\Delta C$ (5)

Where  $\nabla C$  is the concentration gradient. The low diffusion layer thickness and the porous characteristic of the electrodes allow us to assume that gases diffuse in one dimension (in the  $x$  direction; see Fig.1 Hence, the expression of the diffusion flux Eq.  $(5)$  becomes:

$$
J = -D \frac{\partial C}{\partial x}
$$
 (6)

The diffusion coefficient D in porous materials can be calculated according to Giavazzi-Pagano simplified equation:

$$
D = 9700r \frac{\varepsilon}{\tau} \sqrt{\frac{T}{M}}
$$
 (7)

In order to compute  $J$  we use Fick's second law which provides us the variation of the concentration with respect to time:

$$
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \tag{8}
$$

For the solution of this equation, as in [11], we use the Laplace transforms which appears to be more suitable for the state-space representation. Indeed, the Laplace transforms of Eq.  $(8)$  is given by :

$$
\frac{\partial^2 C(x,s)}{\partial x^2} - \frac{s}{D}C(x,s) = 0
$$
\n(9)

With the following boundary conditions:

$$
J^{b}(s) = -AD \frac{\partial C(x, s)}{\partial x}\Big|_{x=0}
$$
 (10)

$$
C^{tpb}(x,s) = C(x,s)|_{x=L}
$$
 (11)

Hence, the solution of Eq.  $(9)$  with respect to these boundary conditions is as follows:

$$
C(x,s) = \frac{J^b(\exp[-\sqrt{s/D}(x-2L)]-\exp[\sqrt{s/D}x]}{AD\sqrt{s/D}[1+\exp[2L\sqrt{s/D}]]} + \frac{C^{pcb}(\exp[\sqrt{s/D}(x+L)]+\exp[-\sqrt{s/D}(x-L)]}{[1+\exp[2L\sqrt{s/D}]]} \qquad (12)
$$

On the surface of the diffusion layer, mass flow rate  $(s)$  $(x,s)$  $\overline{\mathbf{x}}$   $\mathbf{z}$   $\mathbf{z}$ *tpb*  $J^{vpb}(s) = -AD \frac{\partial C(x, s)}{\partial x}\Big|_{x=a}$  $=-AD \frac{\partial C(x,s)}{\partial s}\Big|_{s=L}$  before equation (12) we

have:

$$
\frac{\partial C(x,s)}{\partial x} = -\frac{J^b \left(\exp\left[-\sqrt{s/D}\left(x-2L\right)\right] - \exp\left(\sqrt{s/D}\,x\right]\right)}{AD\left[1 + \exp\left(2L\sqrt{s/D}\right)\right]} + \frac{C^{tpb}\sqrt{s/D}\left(\exp\left[\sqrt{s/D}\left(x+L\right)\right] + \exp\left[-\sqrt{s/D}\left(x+L\right)\right]\right)}{\left[1 + \exp\left(2L\sqrt{s/D}\right)\right]}
$$
(13)

The derivative expression of concentration in zone TPB is:

$$
\frac{\partial C(x,s)}{\partial x}\Big|_{x=L} = \frac{-2J^b(s) \exp(\sqrt{s/D}L)}{AD(1+\exp(2L\sqrt{s/D}))} + \frac{C^{pb}\sqrt{s/D}(\exp(2L\sqrt{s/D})-1)}{(1+\exp(2L\sqrt{s/D}))}
$$
(14)

Then:

$$
Jtpb(s) = \frac{2Jb \exp(L\sqrt{s/D})}{1 + \exp(2L\sqrt{s/D})} -
$$

$$
\frac{Ctpb AD\sqrt{s/D} \exp(2L\sqrt{s/D}) - 1]}{1 + \exp(2L\sqrt{s/D})}
$$
(15)





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$$
\dot{J}^{p b} = \dot{J}^b - \frac{D}{L^2} J^b + \frac{D}{L^2} J^b + \frac{AD}{L} \dot{C}^{p b} \tag{16}
$$

Now, assume that gases are ideal. Then we have:

$$
PV = NRT \Rightarrow P = \frac{NRT}{V} = CRT \tag{17}
$$

Where N is the number of mole of gas present on the volume V. At TPB we have  $x=L$  (see Fig. 1).this infers that

$$
P^{tpb}(s) = RTC(x, s)|_{x=L}
$$
 (18)

The differential equation between mass flow rates and pressures of different gas at TPB and gas bulk :

$$
\dot{J}^{tpb} = \dot{J}^b - \frac{D}{L^2} J^b + \frac{D}{L^2} J^{tpb} + \frac{AD}{RTL} \left( \dot{P}^{tpb} - \frac{P^{tpb}}{T} \dot{T} \right) (19)
$$

Eq. (18) is derived with respect to time is obtained :

$$
\dot{P}^{tpb} = R\dot{T}C^{tpb} + RT\dot{C}^{tpb} = \frac{\dot{T}}{T}P^{tpb} + RT\dot{C}^{tpb} \qquad (20)
$$

By multiplying Eq.  $(14)$  per  $(RT)$  and taking into account with Eq.  $(18)$  and Eq.  $(20)$  it is then:

$$
\dot{P}^{pp} = \frac{\dot{T}}{T} P^{pp} - \frac{2RT}{LA} J^{pp} + \frac{2D}{L^2} P^b - \frac{2D}{L^2} P^{pp} \tag{21}
$$

The substitution of Eq.  $(21)$  in Eq.  $(19)$  gives:

$$
\dot{J}^{tpb} = \dot{J}^b - \frac{D}{L^2} J^b - \frac{D}{L^2} J^{tpb} + \frac{2AD}{RTL^3} P^b - \frac{2AD}{RL^3T} P^{tpb} \tag{22}
$$

## *B. Electrical approach*

The electrochemical phenomena produced inside the fuel cell components are modeled by equivalent circuit. It represents the losses and its dynamic characteristics affect the dynamic behaviors of voltage and current. More precisely, it is composed of three types of resistances representing the ohmic resistance  $(R_m, R_a, R_c)$ activation resistance  $(R_{acta}, R_{actc})$  and the concentration resistance  $(R_{cta}, R_{ctc})$  and two charge capacities  $(C_{dla}, C_{dlc})$  due to the double layer phenomena and one geometrical capacity *Cgeom* between the electrodes.

A typical equivalent circuit which takes into account all these components we obtain the equivalent circuit of the PEMFC shown in Fig. 2, where  $i_c$  is the current of the geometrical capacity[12].



Fig. 2. Equivalent circuit of one cell

Let  $V_e = V_a = V_c$  from the equivalent circuit is as follows:

$$
V_e = -\left(R_{act} + R_{ct}\right)\left[i_t + C_{dl}\frac{dV_e}{dt}\right] \tag{23}
$$

Where:

$$
R_{act} = \frac{A}{i_t} \ln\left(\frac{i_t}{i_o}\right), R_{conc} = \frac{B}{i_t} \ln\left(1 - \frac{i_t}{i_t}\right)
$$
  

$$
i_t = 2FJ_{H_2}^b = 4FJ_{O_2}^b = 2FJ_{H_2O}^b
$$

This implies that:

$$
\frac{dV_e}{dt} = \frac{a_1 J_{H_2}^b V_e}{T \ln \left( a_2 J_{H_2}^b - a_3 J_{H_2}^b \right)} - a_4 J_{H_2}^b \tag{24}
$$

Where 
$$
a_1 = \frac{4\alpha F^2}{RC_{dl}}
$$
,  $a_2 = \frac{2F}{i_o}$ ,  $a_3 = \frac{4F^2}{i_o i_l}$  and  $a_4 = \frac{2F}{C_{dl}}$ 

With:  $V_o = -i_t R_o$ Where:

where:  

$$
\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

$$
R_o = \left[ R_a + R_c + \frac{L_m}{A(a\lambda - b)\exp\left[c\left(\frac{1}{303} - \frac{1}{T}\right)\right]} \right]
$$

Therefore, the voltage and current outputs are calculated as follows:

$$
V_{out} = E + 2V_e + V_o \tag{25}
$$

*F*

2

4

 $\overline{\phantom{a}}$ 

$$
i = i_t - i_c = 2FJ_{H_2}^b + C_{geom} \frac{dV_{out}}{dt}
$$
 (26)

Thus, the current output *i* becomes:

$$
i = 2FJ_{H_2}^b \left( G_2 - 2FC_{geom} \frac{dJ_{H_2}^b}{dt} R_o \right) + G_1
$$
 (27)



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Where:

$$
G_{1} = -C_{geom} \left( \frac{R}{2F} \left[ \ln \left( \frac{P_{H_{2}}^{ppb} \sqrt{P_{O_{2}}^{ppb}}} {P_{H_{2}O}^{ppb}} \right) \frac{dT}{dt} + \frac{1}{2} \frac{dV_{e}}{dt} \right] \right)
$$
  
\n
$$
G_{2} = \left[ 1 - \frac{R_{m}C_{geom}}{(a\lambda - b)} \left( \frac{a\lambda}{dt} + \frac{c(a\lambda - b)}{P_{H_{2}}^{ppb}} \frac{P_{H_{2}}^{ppb}}{P_{H_{2}}^{ppb}} \right) \frac{dT}{dt} \right]
$$

## *C. State-space representation*

In this part, we present the model based on diffusional and electrical approaches in the state-space form. We consider *b*  $H_2O$ *b O b*  $J_{H_2}^b = 2J_{O_2}^b = J_{H_2O}^b$  then we present the diffusional model:

## *1) Hydrogen consumption rate :*

$$
\dot{J}_{H_2}^{p b} = \dot{J}_{H_2}^b - \alpha_3 J_{H_2}^b - \alpha_3 J_{H_2}^{p b} + \frac{\alpha_4}{T} P_{H_2}^b - \frac{\alpha_4}{T} P_{H_2}^{p b} \tag{28}
$$

And its partial pressure in the vicinity of anode at TPB:

 $\mu_a, \mu_2 = \angle D_{H_2} / L_a$ 

$$
\dot{P}_{H_2}^{pbb} = \frac{\dot{T}}{T} P_{H_2}^{pbb} - \alpha_1 T J_{H_2}^{pbb} + \alpha_2 P_{H_2}^b - \alpha_2 P_{H_2}^{pbb}
$$
\n
$$
P_{1} = 2R / AL_a, \alpha_2 = 2D_{H_2} / L_a^2,
$$
\n(29)

Where  $\alpha_1 = 2R/A$ 

 $/L_a^2$ ,  $\alpha_4 = 2AD_H^2 / RL_a^3$ .  $L_3 = D_H$ ,  $/L_a^2$ ,  $\alpha_4 = 2AD_H^2$ ,  $/RL_a^3$ 2  $u^{3}$   $u^{2}$  $H$ <sup>*, l*</sup>a,  $\alpha_4 = \angle AD_{H_2}$  /  $KL_a$  $D_{H_2}$  /  $L_a^2$ ,  $\alpha_4 = 2AD_{H_2}^2$  / RL  $= D_{H_+}/L_a^2, \alpha_4 =$ 

 $\alpha_3 = D_H / L_a \alpha$ 

*2) oxygen consumption rate: b*

$$
\dot{J}_{O_2}^{p b} = \frac{\dot{J}_{O_2}^b}{2} - \frac{B_3}{2} J_{O_2}^b - \beta_3 J_{O_2}^{p b} + \frac{\beta_4}{T} P_{O_2}^b - \frac{\beta_4}{T} P_{O_2}^{p b} \tag{30}
$$

2

And its partial pressure in the vicinity of cathode at TPB:

$$
\dot{P}_{O_2}^{tpb} = \frac{\dot{T}}{T} P_{O_2}^{tpb} - \beta_1 T J_{O_2}^{tpb} + \beta_2 P_{O_2}^{b} - \beta_2 P_{O_2}^{tpb} \tag{31}
$$

Where  $2R/AL_c, \beta_2 = 2D_{0c}/L_c^2$  $B_1 = 2R/AL_c, \beta_2 = 2D_{O_2}/L_c^2$ 2  $L_c$ ,  $p_2 = 2D_{O_2} / L_c$  $R / AL_c$ ,  $\beta_2 = 2D_{Q_c} / L$  $=2R/AL_c, \beta_2=$  $\beta_1 = 2R/AL_c, \beta_2$ 

 $/L_c^2$ ,  $\beta_4 = 2AD_{Q_2}^2/RL_c^3$ .  $B_3 = D_{O_2} / L_c^2$ ,  $\beta_4 = 2AD_{O_2}^2 / RL_c^3$ 2  $\cdots$   $\cdots$   $\cdots$  $Q_0 / L_c$ ,  $p_4 = \angle A D_{O_2} / R L_c$  $D_{O_2}$  /  $L_c^2$ ,  $\beta_4 = 2AD_{O_2}^2$  / RL  $= D_{\Omega} / L_c^2$ ,  $\beta_4 =$  $\beta_3 = D_{O_2}/L_c^2, \beta_4$ 

*3) water consumption rate:*

$$
J_{H_2O}^{ppb} = J_{H_2O}^b - \gamma_3 J_{H_2O}^b - \gamma_3 J_{H_2O}^{ppb} + \frac{\gamma_4}{T} P_{H_2O}^b - \frac{\gamma_4}{T} P_{H_2O}^{ppb} \tag{32}
$$

And its partial pressure in the vicinity of cathode at TPB:

$$
\dot{P}_{H_2O}^{tpb} = \frac{\dot{T}}{T} P_{H_2O}^{tpb} - \gamma_1 T J_{H_2O}^{tpb} + \gamma_2 P_{H_2O}^b - \gamma_2 P_{H_2O}^{tpb}
$$
\n
$$
\gamma_1 = 2R / AL_c, \gamma_2 = 2D_{H_2O} / L_c^2,
$$
\n(33)

2

Where

$$
\gamma_3 = D_{H_2O} / L_c^2, \gamma_4 = 2AD_{H_2O}^2 / RL_c^3.
$$

The above equation show that gases consumption rates depend on input derivatives  $(\dot{u}_1, \dot{u}_5, \dot{u}_6)$  this fact is generally negative because the derivative mode can amplify the outputs noise. Therefore, we use a low-pass filter to eliminate the noise increasing in derivative mode. Consequently, the first order derivative of the input variables can be approximated by the following equation [12]:

$$
sU(s) \approx K \left(1 - \frac{1}{1 + s/K}\right) U(s) \tag{34}
$$

Where  $K$  is an approximation factor greater than 10. Let  $\nu$  be an auxiliary input defines as  $v(s) = (1/(1 + s/K))U(s)$  .thus:

$$
\dot{u} = Ku - v \tag{35}
$$

$$
\dot{\nu} = K^2 u - K \nu \tag{36}
$$

Hence, after introducing the intermediate variables  $v_{H_2}$ ,  $v_T$ ,  $v_\lambda$  to the model, we define the state-space vectors as follows:

$$
x = \left[ J_{H_2}^{qbb}, J_{Q_2}^{qbb}, J_{H_2O}^{qbb}, P_{H_2}^{qbb}, P_{Q_2}^{qbb}, P_{H_2O}^{qbb}, U_{H_2}^{qbb}, U_T, V_e, U_\lambda \right]
$$

This together with all the above equations lead to following state-space model:

$$
\dot{x}_1 = \frac{(Ku_6 - x_8)x_1}{u_6} - \alpha_2 x_1 - \alpha_1 x_4 u_6 + \alpha_2 u_2
$$
\n
$$
\dot{x}_2 = \frac{(Ku_6 - x_8)x_2}{u_6} - \alpha_2 x_2 - \beta_1 x_5 u_6 + \beta_2 u_3
$$
\n
$$
\dot{x}_3 = \frac{(Ku_6 - x_8)x_3}{u_6} - \gamma_2 x_3 - \gamma_1 x_6 u_6 + \gamma_2 u_4
$$
\n
$$
\dot{x}_4 = \frac{-\alpha_4 x_1}{u_6} - \alpha_3 x_4 - x_7 + (K - \alpha_3) u_1 + \frac{\alpha_4 u_2}{u_6}
$$
\n
$$
\dot{x}_5 = \frac{-\beta_4 x_2}{u_6} - \beta_3 x_5 - x_7 + (K - \frac{\beta_3}{2}) u_1 + \frac{\beta_4 u_3}{u_6}
$$
\n
$$
\dot{x}_6 = \frac{-\gamma_4 x_3}{u_6} - \gamma_3 x_6 - x_7 + (K - \gamma_3) u_1 + \frac{\gamma_4 u_4}{u_6}
$$
\n
$$
\dot{x}_7 = K^2 u_1 - Kx_7
$$
\n
$$
\dot{x}_8 = K^2 u_6 - Kx_8
$$
\n
$$
\dot{x}_9 = -\frac{a_1 u_1 x_9}{u_6 \ln(a_2 u_1 - a_3 u_1^2)} - a_4 u_1
$$
\n
$$
\dot{x}_{10} = K^2 u_5 - Kx_{10}
$$

The output equations are presented as follows:

$$
y_1 = V_{out}, y_2 = i
$$

## IV.SIMULATION RESULTS

In this section, we will present the simulation results that demonstrate the effect of the hydrogen flow rate, the gas pressures change and the effects of humidification.



We will simulate the model according to the parameters of [11,12]. Most of these data are taken from [11].The simulation results represent the dynamic behavior for one cell of PEMFC.

## *A. Effect of the hydrogen flow rate:*

The hydrogen flow rate in gas bulk is usually imposed by the conductor of engine. However, we consider it is very important input in order to view the effect of its change on the outputs. We will presents its effects by simulating the responses to pulse hydrogen flow rate change as well as the effect of the disturbance figs (3-5).





Fig. 4 Voltage response to pulse hydrogen flow rate changes



Fig. 5 Current response to pulse hydrogen flow rate changes

One can immediately observe that the variation of the voltage is opposite to the variation of the hydrogen flow rate while the output current change is proportional to it.

## *B. Effect of partial pressures :*

Usually, the control of the fuel cell system is not based on reactant pressure action, because this parameter does not have big influence on the outputs. This fact will be shown in this part where we simulate the effect of hydrogen and water pressures by simulating the output responses to ramp pressure change. The effect of oxygen partial pressure is the same with the effect of hydrogen partial pressure. The simulations results in figs  $(6-11)$ .



Fig. 6 ramp hydrogen partial pressure changes



Fig. 7 Voltage response to ramp hydrogen partial pressure changes



Fig. 8 Current response to ramp hydrogen partial pressure changes



Fig.14 Current response to sinusoidal humidification rate changes

#### V. CONCLUSIONS

In this work, the dynamic model of PEM fuel cell dedicated for transport application are presented with a focus on the diffusional and electrical approaches. We have analyzed the influence of fuel cell operating parameters

## *C. Effect of membrane humidification rate:*

pressure.

The effect of the membrane humidity is important because it influences the loss behavior. Indeed, the membrane humidity affects its resistance value, which is the main element that determines the Ohmic loss. The simulation



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(hydrogen flow rate, partial pressure and membrane humidification rate) on fuel cell performance, we have found that hydrogen flow rate has large effect on output voltage, the output current is generally affected by the hydrogen flow rate, However the membrane humidity and partial pressure does not have big influence on the outputs.

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